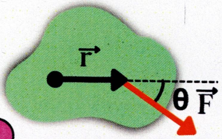
ROTATIONAL MOTION

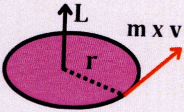
Rotational motion is the motion of a body around a fix axis (see types of motion). Variables of motion in case of rotational motion are

Angular Displacement (θ)	Angular Velocity (ω)
Angular Acceleration (α)	Translational Motion

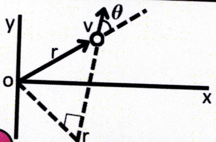
If a body is executing rotation with constant acceleration, the equations of motion can be written as

$\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 - {\omega_0}^2 = 2\alpha t$	θ: angular displacement its unit is radian ω_0:initial angular velocity its unit is rad s^{-1} ω: final angular velocity its unit is rad s^{-1} α: angular acceleration its unit is rads^{-2}

Centre of Mass of System of n Particles	$r_{CM} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum m_i}$
Moment of Inertia	$I = \sum_{i=1}^{n} m_i {r_i}^2$
Torque $\tau = r \times F = rF\sin\theta$	

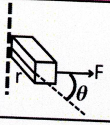
Angular Momentum	**Conservation of Angular Momentum**
$L = I\omega = r \times mv$ 	If the external torque acting on a system is zero, then its angular momentum remains constant. $I_1\omega_1 = I_2\omega_2$
Total kinetic energy of a rolling object Energy $= \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$	
Angular Velocity	$\omega = \frac{d\theta}{dt}, \vec{v} = \omega \times \vec{r}$
Work done by Torque	$\vec{\tau}\, d\vec{\theta} =$ Change in K.E of rotation
Instantaneous rotation power	$P_r = \vec{\tau}.\vec{\omega}$

Angular momentum of particle

$\vec{L}_{CM} = \vec{r} \times (m\vec{v})$ i.e $\vec{L}_T = \vec{L}_0 + \vec{L}_{CM}$ $\vec{L}_{axis} = I_{bodyaxis}\, \vec{\omega}$	

Torque

$\vec{\tau} = \vec{r} \times \vec{F} = Fr\sin\theta$



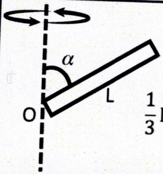
Relation b/w Torque & Angular momentum	$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$
Angular Acceleration	$\alpha = \frac{d\omega}{dt}$ $\quad \vec{a} = \alpha \times \vec{r}$

Torque		Kinetic Energy
$\sum_{i=1}^{n} Fi = 0$	$\sum_{i=1}^{n} \tau i = 0$	$K = \frac{1}{2} I\omega^2$

Moment of Inertia of some bodies

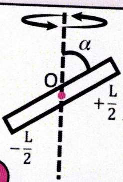
$$I = \sum m_i^2 r_i^2 = \int dmr^2$$

Half Road

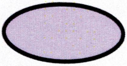
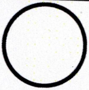
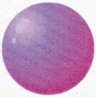


$\frac{1}{3}ML^2\sin^2\alpha$

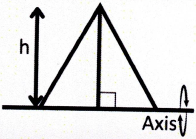
Full Road



$\frac{1}{12}ML^2\sin^2\alpha$

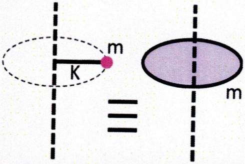
	Ring	MR^2
	Disc	$\frac{1}{2}MR^2$
	Hollow Cylinder	MR^2
	Solid Cylinder	$\frac{1}{2}MR^2$
	Hollow Cone	$\frac{1}{2}MR^2$
	Solid Cone	$\frac{3}{10}MR^2$
	Hollow Sphere	$\frac{2}{3}MR^2$
	Solid Sphere	$\frac{2}{5}MR^2$

Triangular Lamina



$$I = \frac{1}{6}Mh^2$$

Radius of Gyration



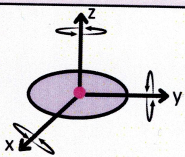
$$I = mK^2$$

$$\text{where } mk^2 = \frac{1}{2}mR^2$$

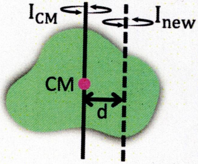
$$K = \frac{R}{\sqrt{2}}$$

Theorm of MI

Perpendicular axis theorem	Parallel axis theorem

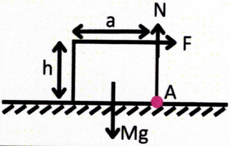


$$I_z = I_x + I_y$$



$$I_{new} = I_{CM} + M_{body}d^2$$

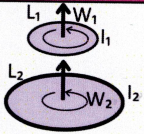
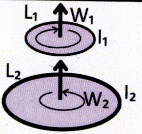
Toppling a body

	$\text{Force} = \dfrac{Mga}{2h}$

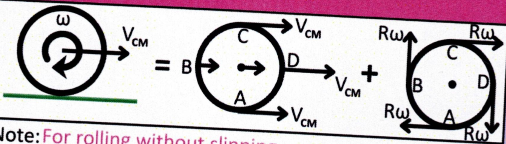
Cases

Condition of slipping before toppling	Condition of toppling before slipping	
$F_{toppling} > F_{slipping}$	$F_{toppling} < F_{slipping}$	
$\dfrac{a}{2h} > \mu_s$	$\dfrac{mga}{2h} < mg\mu_s$	$\dfrac{a}{2h} < \mu_s$

Combination of disc

	
$L_i = I_1 w_1 + I_2 w_2 \quad L_f = (I_1 + I_2)wf$ $W_f = \dfrac{I_1 w_1 + I_2 w_2}{I_1 + I_2}$ $\text{Loss} = \dfrac{1}{2}\dfrac{I_1 I_2}{(I_1 + I_2)}(w_1 - w_2)^2$	$w_f = \dfrac{I_2 w_1 - I_1 w_1}{I_1 + I_2}$ $\text{Loss} = \dfrac{1}{2}\dfrac{I_1 I_2}{(I_1 + I_2)}(w_1 + w_2)^2$

Rolling Motion



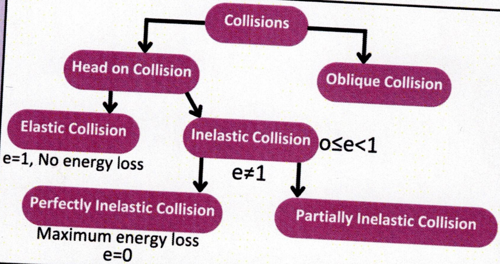
Note: For rolling without slipping

Relative velocity of point A w.r.t. grd = 0

Condition for pure rolling = $\upsilon_{CM} = R_W$

Velocity at a point on rim for rolling sphere

$$\upsilon_{net} = \sqrt{\upsilon^2 + \upsilon_{CM}^2 + 2\upsilon\upsilon_{CM}\cos\theta}$$



Coefficient of Restitution/Resilience (e)

$$e = \frac{\text{relative velocity after collision}}{\text{relative velocity before collision}}$$

Collisions

Perfectly Elastic One Dimensional Collision

- When mass of two colliding bodies are equal, then $v_1=u_2$ and $v_2=u_1$
- If second body of same mass is at rest, then after collision, then $v_1=0$ and $v_2=u_1$
- If $m_1<<m_2$ and m_2 is at rest, then, $v_1=-u_1$ and $v_2=0$
- If $m_1>>m_2$ and m_2 is at rest, then, $v_1=u_1$ and $v_2=2u_1$

$$\upsilon_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{(m_1 + m_2)} \qquad v_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{(m_1 + m_2)}$$

Two Dimensional /Oblique Collision

From law of conservation of momentum

$m_1u = m_1v_1\cos\alpha + m_2v_2\cos\beta$	$0 = m_1v_1\sin\alpha - m_2v_2\sin\beta$

$$\text{Also, } \frac{1}{2}m_1u^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Inelastic Collision

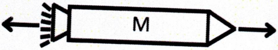
e=coefficient of restitution

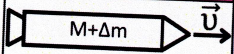
(In perfectly inelastic one dimensional collision e=0)

Loss of Kinetic energy	$\Delta KE = \frac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2(1 - e^2)$
Impulse	$I = \int_{ti}^{tf} \vec{F}.dt = \Delta\vec{p}$

System of variable mass	$M.\frac{d\vec{v}}{dt} = \vec{F}_{ext} + \vec{v}_{rel}\frac{dM}{dt}$

Rocket Propulsion

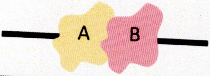




$$V_f - V_i = v_{ext}\ln\frac{M_i}{M_f}$$

Moment of Inertia of Compound Bodies

$$I = I_A + I_B$$



Angular Impulse	$\vec{J} = \int \vec{\tau}_{ext}.\,dt = \int \frac{d\vec{L}}{dt}.\,dt$ $\vec{J} = \Delta\vec{L}$